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SIMPLIFIED DESCRIPTION OF THE FLOW IN A DIAMETRAL DISK FRICTION PUMP

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Using a simplified model, the authors considered the flow in the gap of a diametral disk friction pump; they obtained the parametric dependences of the radial profile of the circular velocity on the circular gradient of pressure and the velocity of the secondary flow. The velocity of the secondary flow was evaluated by the average value, known from experiment, of the circular pressure gradient in the interdisk gap based on the best agreement between these distributions.

Introduction. Disk friction pumps, whose principle of operation is based on the movement of the working medium due to the work of viscous friction forces on moving surfaces, were first described in the works of N. E. Zhukovskii [1] and N. Tesla [2]. In a number of cases, it is preferable to use them instead of pumps of other types owing to such advantages as the low noise level, high anticavitation characteristics, simplicity of the structure, etc. [3, 4]. These apparatuses are finding increasing use in different devices of the flow type, for example, as an element that pumps the medium in a chemical reactor or in an air purifier [5, 6].

This work sought to obtain analytical formulas for the radial distribution of the circular velocity in the interdisk gap of a diametral friction pump based on a simplified model of flow. This model implies that in the interdisk gap, a steady-state fully developed laminar flow of a viscous incompressible fluid occurs. Such relations can be useful for rough evaluations of the influence of different factors on the structure of the flow at the outlet. Unlike the device with a radial canal, in the considered structure the working medium was fed and removed tangentially from the periphery. Axial symmetry of the flow is absent, and there is a circular gradient of pressure in the flow induced by rotating disks. The scheme of the investigated apparatus is shown in Fig. 1. It consisted of a rotor 2 representing a package of disks 6 equidistantly fixed on a shaft using separating rings 7 and placed in a shell 1 having plane-parallel parts (AB and DE) and a cylindrical one (BCD). Placed between the flat components of the shell is a plate 5 separating the incoming 3 and outgoing 4 flows; the plate is parallel to the components of the shell and is set with a small gap from the disks. The structure of the experimental setup made it possible to change the outside R_2 and inside R_1 radii of the disks, the width of the gap b between them, the height of the outlet channel h, and the angular rotational velocity of the shaft Ω. The latter was measured by a strobotachometer. The velocity head and the increase in the total pressure were measured in the outlet cross section by a Pitot tube (for more details, see [7]).

In the design of the considered configuration, the rotating disks, owing to the frictional force, involve the adjacent gas layers into motion. During the movement of the working medium, accelerated by the action of the centrifugal force, from the inlet section of the pump to the outlet section a significant part of it is thrown out of the gap and moves further in the direction opposite to the inlet flow. As a result, a flow similar in many respects to the flow in a curvilinear channel of rectangular cross section with a rotation of the flow by an angle of π is established. However, there are significant differences, too. The flow in the channel is maintained by the negative downstream pressure gradient, and the energy losses are determined by the vis-

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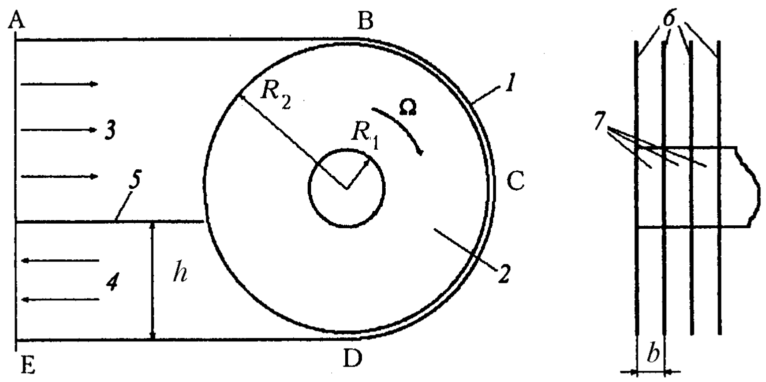

Fig.1. Scheme of a diametral disk friction pump.

cous dissipation near the walls [8, 9]. In the case of our setup, the flow on the segment of rotation BCD, on the contrary, is accelerated due to the work of viscous forces, and the resulting peripheral pressure gradient is positive (there is evacuation at the inlet and injection at the outlet). Thus, pressure forces do not accelerate the flow, but retard it in the main direction. The other difference lies in the direction of radial circulation. In a curvilinear channel, it occurs as a result of the centrifugal force, prevailing in the middle of the channel and forcing the gas to move toward the exterior wall, and in the boundary layers on the side walls – back toward the center of curvature. In our setup, this circulation has a reverse direction: in the middle of the channel the medium flows to the center of rotation, while in the boundary layers on the disks it moves to the exterior wall. The reason is that the highest velocity in the circular direction is developed by the gas in the layers adjacent to the rotating disks. This leads to the prevailing influence here of the centrifugal force, which pushes the medium to the periphery along the walls, and the presence of the shell causes a reverse radial flow in the core of the flow.

As has been indicated above, the problem was considered in a simplified formulation in which the main part of the interdisk gap is assumed to be occupied by a steady-state fully developed laminar flow of a viscous incompressible fluid; the circular pressure gradient induced by the rotation of the disks is constant and has a positive value (a situation inverse to the case of flow in the channel [8], where the negative pressure gradient is the motive force and the energy losses are caused by the forces of viscous friction on the walls). We considered the flow near the middle plane of the interdisk gap that is the plane of symmetry on which the first derivatives of the parameters of the flow over the axial (along the shaft axis) coordinate are equal to zero. In accordance with the results of numerous investigations of disk flows [10–13], the flow was considered as consisting of two components: the main (circular) component and the secondary component (that of radial circulations). Since the thickness of the boundary Ekman layers on the rotating disks under the investigated conditions ($\approx 5\cdot10^{-4}$ m) is small as compared to the width of the gap ($2\cdot10^{-3}$ m or more), the second derivatives with respect to z are approximately equal to zero (according to the data of [10–12], the axial velocity profiles under the indicated condition are close to uniform ones). Under these simplifying propositions, the Navier–Stokes equation for the circular component V of the velocity in dimensionless variables has the form

$$U\frac{dV}{dr}+\frac{UV}{r}=-\frac{C_1}{r}+\frac{1}{\mathrm{Re}}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dV}{dr}\right)-\frac{V}{r^2}\right], \tag{1}$$

where U is the radial component of the velocity made dimensionless by means of ΩR_2; r is the distance along the radius from the center of the disks referred to R_2, and $\mathrm{Re} = R_2^2\Omega/\nu$ is the Reynolds number. In the conducted experiments, the Re number determined in this way did not exceed $1.5\cdot10^5$. Since the laminar-turbulent transition in disk flows occurs for $\mathrm{Re} \approx 3\cdot10^5$ [14, 15], we will further assume that in our experiments we have the laminar regime of flow. The main difference of the fully developed flow in the interdisk gap of a pump with a tangential inlet from the flow in a closed shell lies in the absence of total cylindrical symmetry. This leads to the presence of the nonzero angular pressure gradient (C_1) in Eq. (1).

Equation (1) was solved under two different assumptions regarding the radial dependence of the velocity U of the secondary flow: 1) $U = C_2$ = const; 2) $U(r) = C/r$, where C = const. The second condition corresponds to the exact solution of the equation of continuity $d(rU)/dr = 0$ for a fully developed flow.

Radial Profile of the Circular Velocity for a Constant Velocity of the Secondary Flow ($U = C_2$ = const). In this case, Eq. (1) takes the form

$$\frac{d^2\xi}{dr^2} - \left(\mathrm{Re_s} + \frac{1}{r}\right)\frac{d\xi}{dr} - \alpha = 0 , \tag{2}$$

where $\xi = Vr$; $\mathrm{Re_s} = C_2\mathrm{Re}$ is the Reynolds number of the secondary flow; $\alpha = C_1\mathrm{Re}$.

Using the Lagrangian multiplier method, one, can obtain the following solution of Eq. (2):

$$\xi\,(r) = \int [A + \alpha\,\mathrm{Ei}\,(-\,\mathrm{Re_s}\,r)]\exp\,(\mathrm{Re_s}\,r)\,r dr\,, \quad \mathrm{Ei}\,(z) = \int_{-\infty}^{z} \frac{\exp\,(t)}{t}\,dt\,, \quad z > 0\,,$$

where Ei (z) is the exponential integral function [16]. The first term of the sum under the integration sign can be integrated directly; for the second one, the analytical expression was obtained after the approximation of the combination Ei ($-\mathrm{Re_s}r$) exp ($\mathrm{Re_s}r$)r by polynomials of the third degree using the least-squares method based on the tabular data [16]. As a result, for the radial profile of the circular velocity we have the following equation:

$$V\,(r) = \frac{D}{r} + \frac{A}{r}\exp\,(\mathrm{Re_s}\,r)\,(1 - \mathrm{Re_s}\,r) + \frac{\alpha}{\mathrm{Re_s^2}}\frac{I\,(-\,\mathrm{Re_s}\,r)}{r}\,, \tag{3}$$

where A and D are constants of integration determined by the values of V for two values of the radial coordinate r, and the function $I(z)$ is an approximation (with an accuracy of about 0.1%) of the above integral. Because of the cumbersomeness of the obtained approximational formulas, the latter are not given here.

Radial Profile of the Circular Velocity for U = C/r. The equation for ξ takes the form

$$\frac{C}{r}\frac{d\xi}{dr} = -\,C_1 + \frac{1}{\mathrm{Re}}\left(\frac{d^2\xi}{dr^2} - \frac{1}{r}\frac{d\xi}{dr}\right),$$

where use is made of the above-introduced notation, except $\mathrm{Re_s} = C\,\mathrm{Re}$. By repeating the steps of integration identical to those followed in the derivation of formula (3), we obtain the following expression for the circular velocity:

$$V\,(r) = Ar^k + \frac{D}{r} - \frac{\alpha}{2\mathrm{Re_s}}r\,, \quad k = 1 + \mathrm{Re_s}\,. \tag{4}$$

The constants of integration A and D were calculated from the values of V at two points on the axis r (lying beyond the shear layers near the rigid boundaries).

Comparison of the Velocity Profiles Measured at the Outlet with Theoretical Distributions of the Circular Velocity between the Disks. An attempt was made to apply the obtained formulas (3) and (4) to approximation of the velocity profile of the outlet of the pump and evaluation of the velocity of the sec-

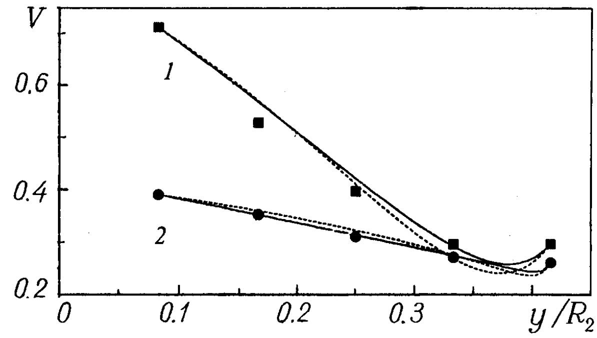

Fig. 2. Velocity profiles: 1) b = 0.002 m, Ω = 960 rpm, and Re = $8.27 \cdot 10^4$; 2) 0.008, 1350, and $1.16 \cdot 10^5$ respectively.

ondary flow. A basis for such an approximation is provided by the fact that the velocity at the pump outlet is formed in motion of the gas in the interdisk gap. A significant feature of the outlet-velocity profiles obtained experimentally [7] and of the radial profiles of the circular velocity calculated from formulas (3) and (4) is their positive bend in the central part closer to the separating plate (correspondingly to the shaft). This bend becomes more pronounced in the calculations as the azimuthal pressure gradient increases and the absolute value of the velocity of the secondary flow decreases. This is attributable to the fact that the pressure force retards the flow more distinctly at smaller distances from the shaft, since the circular velocity and the velocity head are lower here than at the edges of the disks. In turn, the secondary flow is directed toward the center of rotation and transports large values of the circular velocity from the periphery closer to the shaft, and this causes the bend to decrease or to disappear as the intensity of the secondary flow increases.

Based on the indicated qualitative similarity of the circular-velocity distributions over the radius that are calculated from formulas (3) and (4) to the measured profiles of the flow velocity at the outlet of the pump, we evaluate (in order of magnitude) the velocity of the secondary flow between the disks using the average circular pressure gradient obtained experimentally. This was carried out by approximating the calculated profiles, obtained from formulas (3) and (4), to experimental ones by means of variation of the constants C and C_2. An example of the results of such an approximation is presented in Fig. 2, where the points show the measured values of the dimensionless velocity at the pump outlet; the radial distribution of the circular velocity in the interdisk gap calculated from formula (3) is shown as the dashed line, while the distributions calculated from formula (4) are shown as solid lines. On the axis of ordinates, we plotted the dimensionless coordinate y/R_2, where the distance y was counted off from the lower plane of the shell in consideration of experimental curves and from the external edge of the disks in the direction to their center for theoretical curves. The inside and outside radii of the pump disks are equal to 0.06 and 0.12 m respectively. Figure 1 corresponds to the calculations for a width of the interdisk gap of 0.002 m and a rotational velocity of 16 rps, and Fig. 2 corresponds to the calculations for 0.008 and 22.5 respectively. The pressure gradient was calculated based on the experimentally measured pressure difference between the outlet and inlet cross sections, which was averaged throughout the height of the outlet channel [7]. The values of the constants C_2 and C in formulas (3) and (4) for which we observe the closest position of the experimental and calculated curves are equal to: $C_2 = -1.85 \cdot 10^{-4}$ ($\mathrm{Re_s} = -15.3$) and $C = -1.45 \cdot 10^{-4}$ ($\mathrm{Re_s} = -13.7$) for case 1 and $C_2 = -9 \cdot 10^{-4}$ ($\mathrm{Re_s} = -104.6$) and $C = 7 \cdot 10^4$ ($\mathrm{Re_s} = -90.4$) for case 2.

CONCLUSIONS

1. The investigation carried out showed that the presented analysis of the flow between the disks of a diametral pump yields a radial profile of the circular velocity qualitatively similar to the velocity profile at the outlet of the device. By selecting the values of the parameters A, D, and C (or C_2) in formulas (3) and

(4) for which the experimental and calculated velocity profiles show the best agreement, we evaluated the velocity of the secondary flow. It turned out to be much lower (10^4 times in order of magnitude) than the velocity in the main direction of flow; however, its influence on the character of the velocity distribution $V(r)$ is quite substantial.

2. Results of the calculations from formulas (3) and (4) are very sensitive to the values of the circular pressure gradient and the velocity of the secondary flow; consequently, these quantities must be taken into account in calculating and designing devices of the type of diametral disk friction pumps.

NOTATION

A, *C*, and *D*, constants of integration; *U*, radial component of the velocity; *V*, circular component of the velocity; *r*, radial coordinate; *z*, axial coordinate; *R*, radius of the disk; *h*, height of the outlet channel; *b*, width of the interdisk gap; *y*, coordinate along the vertical; Ω, rotational velocity of the disks; ν, kinematic viscosity; Re, Reynolds number; Ei, exponential integral function. Subscripts: 1 and 2, inside and outside radii of the disks; s, secondary flow.

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